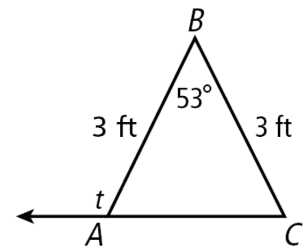
Pre-AP Geometry 12-4 Study Guide: Inscribed Angles (pp 820-824)

<u>Attendance Problems.</u> Find each value.

1. $m\measuredangle BCA$ **2.** t



Solve for x.

3. $58 - x = 4(x + 7)$ **4.** $2(x - 8) = 8$

Pre-AP Geometry 12-4 Study Guide: Inscribed Angles (pp 820-824)

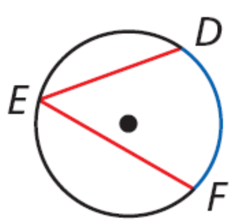
- I can find the measure of an inscribed angle.
- I can use inscribed angles and their properties to solve problems.

Vocabulary		
Inscribed angle	Intercepted arc	Subtend

Common Core

CC.9-12.G.C.2 Identify and describe relationships among inscribed angles, radii, and chords.
CC.9-12.G.C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
CC.9-12.G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. An **intercepted arc** consists of endpoints that lie on the sides of an inscribed angle and all the points of the circle between them. A chord or arc **subtends** an angle if its endpoints lie on the sides of the angle.



$\angle DEF$ is an **inscribed angle**.

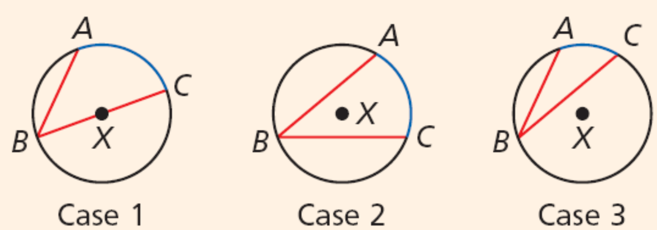
$\overset{\frown}{DF}$ is the **intercepted arc**.

$\overset{\frown}{DF}$ subtends $\angle DEF$.

Theorem 11-4-1 Inscribed Angle Theorem

The measure of an inscribed angle is half the measure of its intercepted arc.

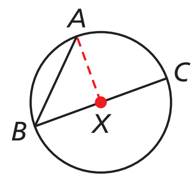
$$m\angle ABC = \frac{1}{2} m\overset{\frown}{AC}$$



Pre-AP Geometry 12-4 Study Guide: Inscribed Angles (pp 820-824)

PROOF

Inscribed Angle Theorem



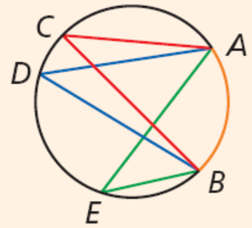
Given: $\angle ABC$ is inscribed in $\odot X$.
Prove: $m\angle ABC = \frac{1}{2}m\overset{\frown}{AC}$

Proof Case 1:

$\angle ABC$ is inscribed in $\odot X$ with X on $\overline{BC}$. Draw $\overline{XA}$. $m\overset{\frown}{AC} = m\angle AXC$.
By the Exterior Angle Theorem $m\angle AXC = m\angle ABX + m\angle BAX$.
Since $\overline{XA}$ and $\overline{XB}$ are radii of the circle, $\overline{XA} \cong \overline{XB}$. Then by definition $\triangle AXB$ is isosceles. Thus $m\angle ABX = m\angle BAX$.

By the Substitution Property, $m\overset{\frown}{AC} = 2m\angle ABX$ or $2m\angle ABC$.
Thus $\frac{1}{2}m\overset{\frown}{AC} = m\angle ABC$.

Corollary 11-4-2

COROLLARY	HYPOTHESIS	CONCLUSION
If inscribed angles of a circle intercept the same arc or are subtended by the same chord or arc, then the angles are congruent.	 $\angle ACB$, $\angle ADB$, and $\angle AEB$ intercept $\overset{\frown}{AB}$.	$\angle ACB \cong \angle ADB \cong \angle AEB$ (and $\angle CAE \cong \angle CBE$)

Paper plate activity.

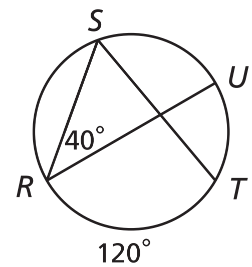
Question: I love my new bracelet, but why does it have the word *angle* written on it?
Answer: It's your very own *inscribed angle*!

"Appreciation can make a day--even change a life. Your willingness to put it into words is all that is necessary."—*Writer, Margaret Cousins*

Pre-AP Geometry 12-4 Study Guide: Inscribed Angles (pp 820-824)

1 Finding Measures of Arcs and Inscribed Angles

Find each measure.



A $m\angle RST$

$m\angle RST = \frac{1}{2} m\overset{\frown}{RT}$ *Inscribed ∠ Thm.*

$= \frac{1}{2}(120°) = 60°$ *Substitute 120 for $m\overset{\frown}{RT}$.*

B $m\overset{\frown}{SU}$

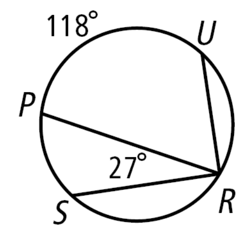
$m\angle SRU = \frac{1}{2} m\overset{\frown}{SU}$ *Inscribed ∠ Thm.*

$40° = \frac{1}{2} m\overset{\frown}{SU}$ *Substitute 40 for m∠SRU.*

$m\overset{\frown}{SU} = 80°$ *Mult. both sides by 2.*

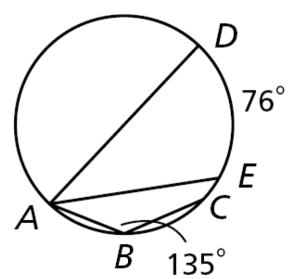
Example 1. Find each measure.

A. $m\measuredangle PRU$ **B.** $m\overset{\frown}{SP}$



Guided Practice. Find each measure.

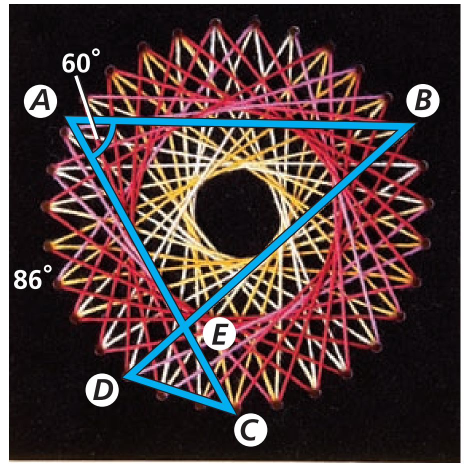
5. $m\overset{\frown}{ADC}$ **6.** $m\measuredangle DAE$



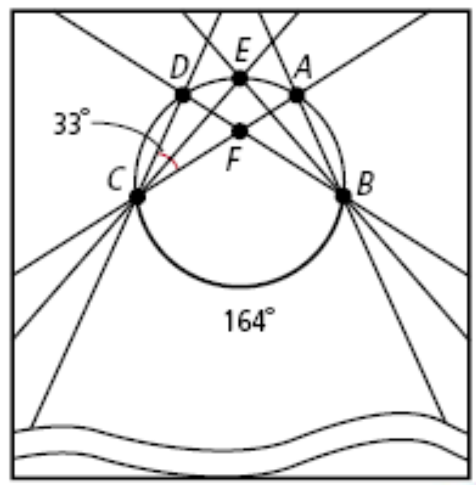
2 Hobby Application

Find $m\angle DEC$, if $m\overparen{AD} = 86°$.

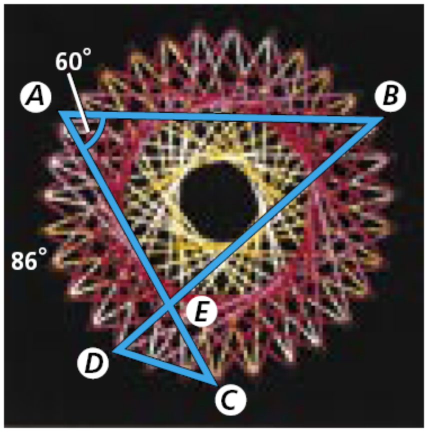
$\angle BAC \cong \angle BDC$	*$\angle BAC$ and $\angle BDC$ intercept $\overparen{BC}$.*
$m\angle BAC = m\angle BDC$	*Def. of $\cong$*
$m\angle BDC = \mathbf{60°}$	*Substitute 60 for $m\angle BDC$.*
$m\angle ACD = \frac{1}{2} m\overparen{AD}$	*Inscribed $\angle$ Thm.*
$= \frac{1}{2}(\mathbf{86°})$	*Substitute 86 for $m\overparen{AD}$.*
$= 43°$	*Simplify.*
$m\angle DEC + \mathbf{60} + \mathbf{43} = 180$	*$\triangle$ Sum Theorem*
$m\angle DEC = 77°$	*Simplify.*



Example 2. An art student turns in an abstract design for his art project. Find $m\measuredangle DFA$.

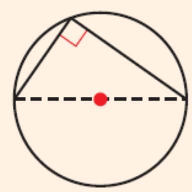


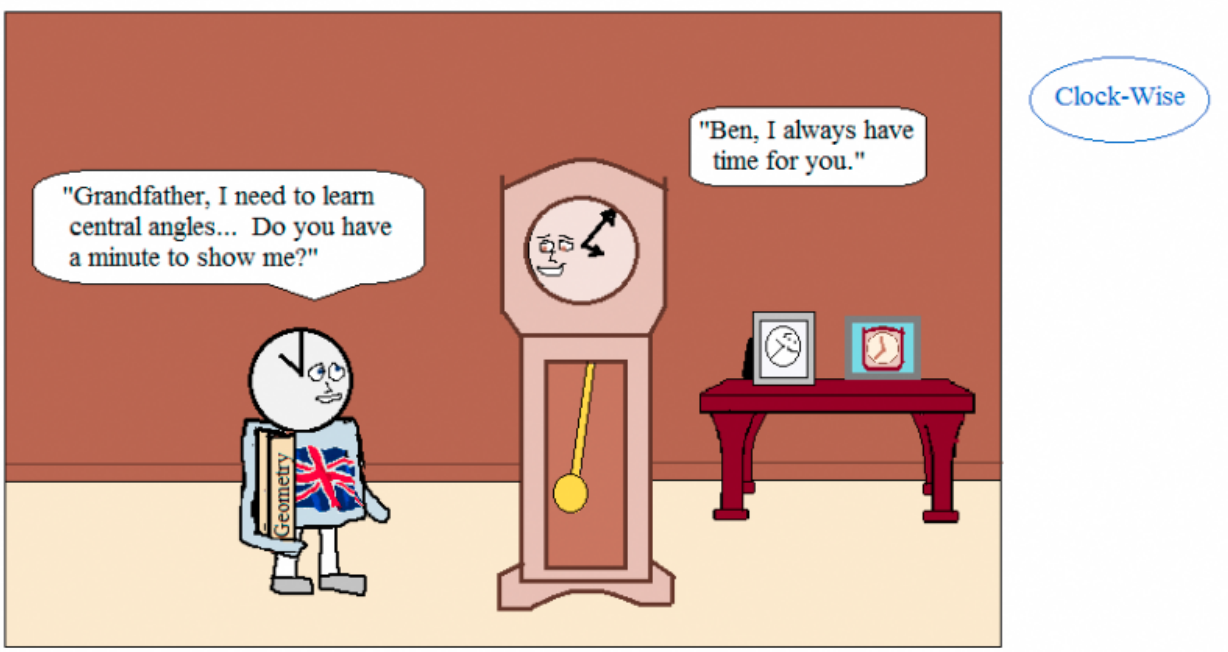
7. <u>Guided Practice</u>. Find $m\measuredangle ABD$ & $m\overset{\frown}{BC}$ in the string art.



Theorem 11-4-3

An inscribed angle subtends a semicircle if and only if the angle is a right angle.





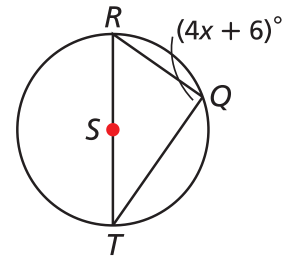
The old-timer demonstrates acute angles --- without being obtuse.

LanceAF #85 5-10-13
www.mathplane.com

3 Finding Angle Measures in Inscribed Triangles

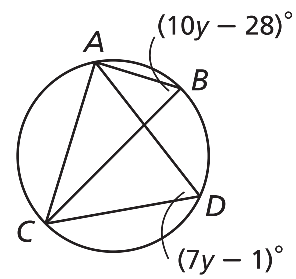
Find each value.

A x



$\angle RQT$ is a right angle	*$\angle RQT$ is inscribed in a semicircle.*
$m\angle RQT = 90°$	*Def. of rt. $\angle$*
$\mathbf{4x + 6} = 90$	*Substitute $4x + 6$ for $m\angle RQT$.*
$4x = 84$	*Subtract 6 from both sides.*
$x = 21$	*Divide both sides by 4.*

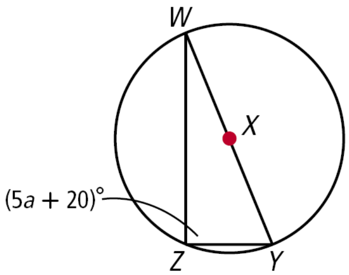
B $m\angle ADC$



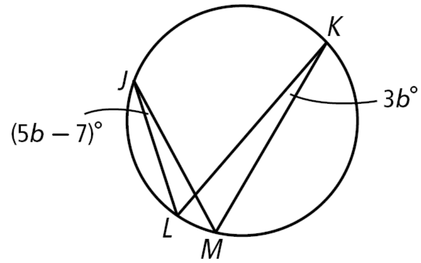
$m\angle ABC = m\angle ADC$	*$\angle ABC$ and $\angle ADC$ both intercept $\overset{\frown}{AC}$.*
$\mathbf{10y - 28} = \mathbf{7y - 1}$	*Substitute the given values.*
$3y - 28 = -1$	*Subtract $7y$ from both sides.*
$3y = 27$	*Add 28 to both sides.*
$y = 9$	*Divide both sides by 3.*
$m\angle ADC = 7\mathbf{(9)} - 1 = 62°$	*Substitute 9 for y.*

Example 3. **Find the value of each.**

A. a



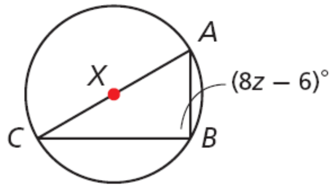
B. $m\measuredangle LJM$



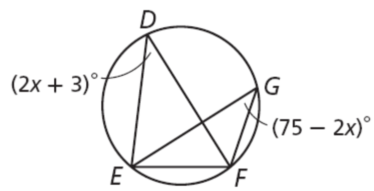
Pre-AP Geometry 12-4 Study Guide: Inscribed Angles (pp 820-824)

Guided Practice. Find the value of each.

8. z

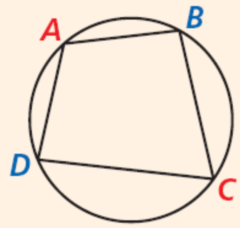


9. $m\measuredangle EDF$

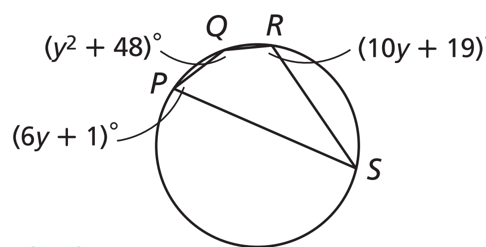


12-4 Inscribed Angles (p 824) 13, 15, 16, 17, 19.

Theorem 11-4-4

THEOREM	HYPOTHESIS	CONCLUSION
If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.	 *ABCD* is inscribed in $\odot E$.	$\angle A$ and $\angle C$ are supplementary. $\angle B$ and $\angle D$ are supplementary.

4 Finding Angle Measures in Inscribed Quadrilaterals



Find the angle measures of *PQRS*.

Step 1 Find the value of y.

$m\angle P + m\angle R = 180^\circ$ — *PQRS is inscribed in a $\odot$.*

$6y + 1 + 10y + 19 = 180$ — *Substitute the given values.*

$16y + 20 = 180$ — *Simplify.*

$16y = 160$ — *Subtract 20 from both sides.*

$y = 10$ — *Divide both sides by 16.*

Step 2 Find the measure of each angle.

$m\angle P = 6(10) + 1 = 61^\circ$ — *Substitute 10 for y in each expression.*

$m\angle R = 10(10) + 19 = 119^\circ$

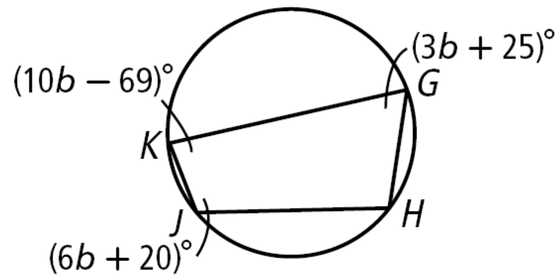
$m\angle Q = 10^2 + 48 = 148^\circ$

$m\angle Q + m\angle S = 180^\circ$ — *$\angle Q$ and $\angle S$ are supp.*

$148^\circ + m\angle S = 180^\circ$ — *Substitute 148 for $m\angle Q$.*

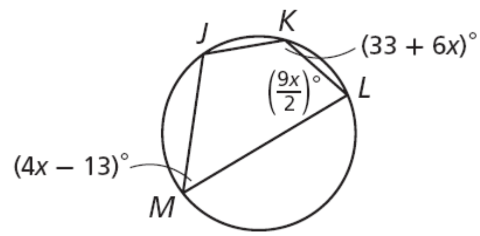
$m\angle S = 32^\circ$ — *Subtract 148 from both sides.*

<u>Example 4</u>. Find the angle measures of *GHJK*.



Pre-AP Geometry 12-4 Study Guide: Inscribed Angles (pp 820-824)

10. <u>Guided Practice</u>. Find the angle measures of *JKLM*.



12-4 Inscribed Angles (p 824) 13, 15, 16, 17, 19, 21-26, 29, 34, 35, 37, 39, 40.
Question: I love my new bracelet, but why does it have the word *angle* written on it?
Answer: It's your very own *inscribed angle*!